

## Matlab/Freemat: Matrix Eigenvalues and Eigenvectors

That is the eigenvalues and eigenvectors of a matrix  $A$  are the non-trivial vectors  $\underline{x}$  and scalars  $\lambda$  that satisfy the following equation:

$$A \underline{x} = \lambda \underline{x}.$$

In Freemat/Matlab, the eigenvalues of the matrix - or the eigenvalues and eigenvectors of a matrix - can be found using the eig command.

### Example 1. $2 \times 2$ matrix

For example the solution of the eigenvalue problem with  $A = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$  can be determined with the following code.

```
--> A=[3 -2; -1 4]
A =
3 -2
-1 4
-> eig(A)
ans =
2
5
-> [E,V]=eig(A)
E =
-0.8944  0.7071
-0.4472 -0.7071
V =
2 0
0 5
```

Note that the command eig(A) simply returns the eigenvalues of  $A$ , but the command [E,V]=eig(A) returns both the eigenvalues and eigenvectors. The eigenvalues are given by the diagonal components of the matrix  $V$  and the corresponding eigenvectors are the columns of the matrix  $E$ .

Comparing the results with the document from which the examples came<sup>1</sup>, we see that the eigenvalue results match, but the eigenvector results do not. The eigenvalues found in the referenced document are  $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ . However, with eigenvectors it is the ratio of the elements that matters, not the actual values. In Matlab/Freemat, the eigenvectors have been normalised<sup>2</sup>, so that their norm is one.

### Example 2. $3 \times 3$ matrix

For example the solution of the eigenvalue problem with  $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 1 \\ -2 & 1 & -1 \end{pmatrix}$  can be determined with the following code.

```
--> A=[1 -1 0; 1 2 1; -2 1 -1]
A =
1 -1 0
1 2 1
-2 1 -1
--> eig(A)
ans =
-1.0000
1.0000
2.0000
--> [E,V]=eig(A)
E =
-0.1361 -0.7071 -0.5774
0.2722 -0.0000 0.5774
0.9526 0.7071 0.5774
V =
-1.0000 0 0
0 1.0000 0
0 0 2.0000
```

### Example 3. Generalised Eigenvalue Problem with $2 \times 2$ Matrices;

Matlab/freemat also have the facility to solve the generalised eigenvalue problem

$$A \underline{x} = \lambda B \underline{x}.$$

For example the following code can be used to solve the generalised eigenvalue problem

$$\begin{pmatrix} 0 & 2 \\ 4 & 2 \end{pmatrix} \underline{x} = \lambda \begin{pmatrix} -2 & 3 \\ 0 & 5 \end{pmatrix} \underline{x}.$$

```
--> A=[0 2; 4 2]
A =
0 2
4 2
--> B=[-2 3; 0 5]
B =
-2 3
0 5
--> eig(A,B)
ans =
0.8000 + 0.4000i
0.8000 - 0.4000i
--> [E,V]=eig(A,B)
E =
0.4037 + 0.5000i  0.4037 - 0.5000i
0.9037 + 0.0963i  0.9037 - 0.0963i
V =
0.8000 + 0.4000i    0
0          0.8000 - 0.4000i
```